FREDERICK S. ELLETT, JR. AND DAVID P. ERICSON

CORRELATION, PARTIAL CORRELATION, AND CAUSATION*

ABSTRACT. Philosophers and scientists have maintained that causation, correlation, and "partial correlation" are essentially related. These views give rise to various rules of causal inference. This essay considers the *claims* of several philosophers and social scientists for causal systems with dichotomous variables. In section 2 important commonalities and differences are explicated among four major conceptions of correlation. In section 3 it is argued that whether correlation can serve as a measure of A's causal influence on B depends upon the conception of causation being used and upon certain background assumptions. In section 4 five major kinds of "partial correlation" are explicated, and some of the important relations are established among two conceptions of "partial correlation", the conception of "screening off", the conception of "partitioning", and the measures of causal influence which have been suggested by advocates of path analysis or structural equation methods. In section 5 it is argued that whether any of these five conceptions of "partial correlation" can serve as a measure of causal influence depends upon the conception of causation being used and upon certain background assumptions.

The important conclusion is that each of the approaches (considered here) to causal inference for causal systems with dichotomous variables stands in need of important qualifications and revisions if they are to be justified.

1. INTRODUCTION

This essay considers a general set of claims of certain philosophers and social scientists that causation, correlation, and "partial correlation" are essentially related. These claims are considered for causal systems with dichotomous variables. Philosophers of science and social scientists, such as H. Blalock (1964), Campbell and Stanley (1963, pp. 64–6), Kendall and Lazarsfeld (1950), Reichenbach (1956, pp. 156–9, 190), and Suppes (1970, pp, 12, 23–5, 28), hold that if A causes B, then A is positively correlated with B. Others, such as H. Asher (1976), P. Bentler (1980), and J. Davis (1975), hold that if A causes B, then a certain conception of correlation can serve as a measure of the degree of A's causal influence on B. The views of these researchers give rise to rules of causal inference that can be used, for example, to reject certain causal hypotheses when A is not positively correlated with B. Further conditions must, of course, be added to develop sufficient conditions for A to be a cause of B.

Synthese **67** (1986) 157–173.

Section 2 explicates four conceptions of correlation and draws out some of the important commonalities and differences among them. Section 3 argues that whether a conception of correlation can serve as a measure of *A*'s causal influence on *B* depends upon the conception of causation being used and upon certain background assumptions.

Section 4 explicates the relations among two conceptions of "partial correlation", the conception of "screening off", the conception of "partitioning", and the measures of causal influence that have been suggested by advocates of path analysis or structural equation methods. Certain philosophers and social scientists maintain that "partial correlation" tests can be used to determine, for example, whether a certain variable *B* is a causally intervening variable between *A* and *C*, or whether *B* is a common cause of *A* and *C*. Section 5 argues that whether such views on causal inference can be justified depends upon the conception of causation being used and upon certain background assumptions.

The general conclusion is reached that the various rules of causal inference commonly advocated by these researchers stand in need of serious qualification and revision.

2. CONCEPTIONS OF CORRELATION

In this section we explicate four important conceptions of correlation. Before we begin, however, notice that each of the four conceptions of correlation will be defined for the *population* of events *A* and *B*. We are not interested here in the measurement and statistical issues which are related to using an observed sample to estimate or determine the (true) population correlation. We are concerned only with the (alleged) relations for the *population* correlation. Also notice that Nagel, Kendall and Lazarsfeld, and Salmon are concerned with correlations among events (or attributes), while Asher, Blalock, and Simon are primarily concerned with relations among *quantities*. P. Suppes (1970), on the other hand, has developed a view for each kind of variable. We have included Asher, Blalock, and Bentler here because these authors have argued that the approach developed for quantitative variables can also be used for *ordinal* variables or for attribute (or *category*) variables. Asher argues for extending the approach to ordinal data (pp. 64–6). Those following the so-called Simon-Blalock approach have included attribute variables (e.g., Davis, 1975). Most notable are Blalock's claims that the method can be applied to attribute variables (pp. 32, 71–7, 119–24). Often these authors choose to illustrate the general method by investigating examples which have attribute variables (Blalock, pp. 71–7; Suppes, pp. 117–18). We are concerned with these approaches as they apply to dichotomous variables.

We begin by considering the conception used by the social scientists P. Kendall and P.F. Lazarsfeld (1950, pp. 153–8) and by the philosopher of science, Ernest Nagel (1961, pp. 512–3). (Nagel also uses the second conception as well.) In the first conception, two variables (or states of affairs) A and B are positively correlated if and only if the probability of A and B minus the probability of A times the probability of B is greater than zero. In standard notation, A and B are positively correlated if and only if $P(A.B) - P(A)P(B) > 0$. A and B are negatively correlated if and only if $P(A.B) - P(A)P(B) < 0$; A and B are uncorrelated if and only if $P(A.B)$ equals $P(A)P(B)$. One can easily show that:

(a) $$P(A.B) - P(A)P(B) = P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B).$$

The second conception of correlation is used by philosophers of science Hans Reichenbach (1956) and Patrick Suppes (1970). Two variables are positively correlated if and only if the probability of B given A minus the probability of B is greater than zero. That is, A and B are positively correlated if and only if $P(B/A) - P(B) > 0$. A and B are negatively correlated if and only if $P(B/A) - P(B) < 0$; A and B are uncorrelated if and only if $P(B/A)$ equals $P(A)$. Given that $P(A)$ is nonzero, it can be shown that:

(b) $$P(B/A) - P(B) = [P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B)]/P(A).$$

A third conception of correlation is used by such philosophers of science as Wesley Salmon (1980).[1] Two variables are positively correlated if and only if the probability of B given A minus the probability of B given not A is greater than zero. That is, A and B are positively correlated if and only if $P(B/A) - P(B/\bar{A}) > 0$; A and B are negatively correlated if and only if $P(B/A) - P(B/\bar{A}) < 0$; A and B are uncorrelated if and only if $P(B/A) = P(B/\bar{A})$. Given that $1 > P(A) > 0$, it can be shown that:

(c) $$P(B/A) - P(B/\bar{A}) = [P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B)]/[P(A)P(\bar{A})]$$

The fourth conception of correlation is used by social scientists H. B. Asher (1976), H. M. Blalock, Jr. (1964), and H. A. Simon. Two variables are positively correlated if and only if the Φ (phi) coefficient is positive; A and B are uncorrelated if and only if Φ is negative; and A and B are uncorrelated if and only if Φ is zero. The Φ (phi) correlation is *defined* as follows:

(d) $\Phi_{AB} = [P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B)]$ divided by $[P(A)P(\bar{A})P(\bar{B})P(B)]^{1/2}$.

We are ready to draw out the commonalities and differences among the four conceptions of correlation. Over the domain where all are defined ($1 > P(A) > 0$ and $1 > P(B) > 0$), the conceptions have the same sign, which is given by the numerator term in the right hand side of equations a, b, c, and d:

(1) $P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B)$.

This is the covariance of A and B. It would make no difference which of the conceptions of correlation were used in a *qualitative* theory of causal inference. A qualitative theory concludes that A is not a cause of B if A is not positively correlated with B; and perhaps it finds some weak kind of confirming evidence for A's being a cause of B if A is positively correlated with B. But each of the four conceptions gives the same information about whether A and B are positively correlated.

Things are rather different in the case of a *quantitative* theory of causal inference. A quantitative theory tries to determine not only whether A causes B, but also it tries to determine the degree or magnitude of A's causal influence on B. Many have attempted to develop a quantitative theory so that a particular conception of correlation (in certain situations at least) would give the magnitude of A's causal influence on B. (See Asher[2]; Bentler; and Davis.)[3] There are differences among the four conceptions of correlations with respect to the magnitude of the correlations. Given that $1 > P(A) > 0$ and that $1 > P(B) > 0$, the following relations hold: whenever $P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B)$ is zero, then the conceptions of correlation are all zero; whenever $P(A.B)P(\bar{A}.\bar{B}) - P(A.\bar{B})P(\bar{A}.B)$ is nonzero, then the conceptions are all nonzero and have the same sign, say positive, but their magnitudes will differ. So understood, it can be shown that: (i) $P(B/A) - P(B)$ is greater than $P(A.B) - P(A)P(B)$; (ii) $P(B/A) -$

$P(B/\bar{A})$ is greater than $P(B/A) - P(B)$; and (iii) Φ_{AB} is greater than $P(A.B) - P(A)P(B)$. When the correlations are positive, (iv) Φ_{AB} can be greater than $P(B/A) - P(B/\bar{A})$, (v) Φ_{AB} can be less than $P(B/A) - P(B/\bar{A})$ but greater than $P(B/A) - P(B)$, and (vi) Φ_{AB} can be less than $P(B/A) - P(A)$ but greater than $P(A.B) - P(A)P(B)$.

3. CORRELATION AS A MEASURE OF CAUSAL INFLUENCE

Writers such as Campbell and Stanley, Kendall and Lazarsfeld, Reichenbach, and Suppes maintain that if A is a cause of B, then A and B will be positively correlated. On the other hand, Asher, Blalock, and Davis conclude that if A is a cause of B, then a conception of correlation between A and B will determine the magnitude of the A's causal strength on its effect B. Here we argue that if some effect B has two causes, A and C, then even if A is a sufficient cause of B, or if A is a probabilistic cause of B (with probability 1), in general the correlation between A and B must be nonnegative but it need not equal 1. If A is a probabilistic cause of B (with probability $p < 1$), then the sign and the magnitude of the correlation will depend upon background assumptions involving certain features of the other causes of B.

It will be helpful here to give a brief description of a general kind of causal system. Suppose that there are two distinct coins, where H represents a tossing of the first coin and J represents a tossing of the second coin and where the coin tossings H and J are not causally related to one another. Furthermore, suppose that tossing (H) the first coin has the probability p of producing an outcome (K) of at least one head and that tossing (J) the second coin has the probability r of producing an outcome K of at least one head. If both coins are tossed at the same time, we assume that the probability of getting an outcome (K) of at least one head is $p + r - pr$. We shall say that H is a *probabilistic cause* of an outcome K of at least one head with probability p. A probabilistic cause is taken as an idealized coin toss. In a recent paper (Ellett and Ericson, 1985) which considers N. Cartwright's (1979 and 1983) views on the relationship between causal laws and laws of association, we derive the following: If H is a probabilistic cause of K with probability p, then $P(K/H.G) = p$ and $P(K/\bar{H}.G) = 0$, where G is the hypothetical state of affairs where none of the other causal factors of K, if any exist, are present. The expression $P(K/\bar{H}.G) = 0$ roughly says that if no causes of K are present, then the probability of

K's occurring is zero. In the situation considered above, G becomes $\bar{J}$ and we get $P(K/H.\bar{J}) = p$.

We believe that it can be shown the $P(K/H.G)$ gives a quite intuitive and plausible measure of the magnitude of H's causal influence on K. Notice that if H is (empirically) sufficient for K, then $P(K/H.G) = 1$ and $P(K/\bar{H}.G) = 0$. Nevertheless, if H is sufficient for K, then $P(K/H.G) = 1$, but it also follows that the event $(H.\bar{K})$ cannot occur. Given certain conceptions of probability, it is logically possible for cases of $H.\bar{K}$ to occur even though $P(K/H.G) = 1$.[4] We shall argue that the defensibility of the causal inference rules depends in part on the concept of causation one is employing.

An important characteristic still requires specification for this general causal system (in which the effect K has only the two causes H and J): the background assumption about the joint distribution of the cause H and the cause J needs to be specified. We assume that $1 > P(H) > 0$ and consider three possible situations: (i) H and J are uncorrelated, (ii) H and J have zero expectation (that is, $P(H.J)$ is zero) and (iii) H and J are maximally negatively correlated.

Given this characterization of a general causal system, is it defensible to use correlation as a measure of H's causal influence on K? First, suppose that H is a sufficient cause of K and that H and J are uncorrelated.[5] Then $P(K/H) - P(K/\bar{H})$ equals $1 \cdot [1 - rP(J)]$ so that the correlation between H and K must be *nonnegative*. Furthermore, when $r = 1$, $P(K/H) - P(K/\bar{H})$ will equal one if and only if $P(J) = 0$; its magnitude will be less than one whenever $P(J) > 0$. It can also be shown that none of the other three conceptions of correlation will give the magnitude of one in this situation.[6] Where the expectation of H and J is zero, $P(K/H) - P(K/\bar{H})$ equals $1 - rP(J/\bar{H})$, and its value must be nonnegative with its sign and magnitude depending upon the value of $rP(J/\bar{H})$. The correlation must be nonnegative, but it need not equal 1. Finally, where H and J are maximally negatively correlated, $P(K/H) - P(K/\bar{H})$ equals $1 - r$. Thus, when H is a sufficient cause of K, the correlation between H and K must be *nonnegative*, but in general it will not equal 1. When H is a probabilistic cause of K (with probability 1), it can also be shown that the correlation must be nonnegative, but in general it will not equal 1.

These results show that none of the four correlations can be used to determine the magnitude of H's causal influence on K when H is a sufficient condition for K. For it is quite intuitive and plausible to hold that if H is sufficient for K, then H should be regarded as having the maximal causal influence on K; but none of the four conceptions of correlation guarantees this result. Whether the correlation equals one depends on the features of the other causal factor and how it is distributed in the population. The joint distribution and features of the other causes are important (background) characteristics of the causal system, but they confound the determination of H's causal influence on K. For example, if H is a sufficient cause and J is a sufficient cause, the correlation between H and K will always be less than 1 given that $P(J) > 0$; and the correlation can be zero. Nonetheless, given this conception of causation, the correlation between H and K must be *nonnegative*, and thus it could be used as a qualitative measure.

Many social scientists and philosophers, however, would urge that one take 'A causes B' to mean that A is a probabilistic cause (with a probability of q) of B (where $0 < q < 1$). So let us suppose that H is a probabilistic cause of K with $p < 1$. Where the causes H and J are uncorrelated, $P(K/H) - P(K/\bar{H})$ equals $p(1 - rP(J))$, a nonnegative number, but as long as $rP(J) > 0$, it will not yield the magnitude p. It can also be shown that the other conceptions of correlation face this difficulty as well. This result, however, partially vindicates the claims of such writers as Asher, Blalock, and Bentler, for when the causes H and J are uncorrelated, the correlation between H and K will be nonnegative, and hence it can provide a qualitative measure for A's being a cause of B. But where the expectation of H and J is zero, $P(K/H) - P(K/\bar{H})$ equals $p - rP(J/\bar{H})$; and so the correlation between H and K can be positive, negative, or zero, depending on the value of $rP(J/\bar{H})$. Finally, where H and J are maximally negatively correlated, then $P(K/H) - P(K/\bar{H}) = p - r$; and so the correlation between H and K can be positive, negative, or zero, depending on the value of r.

Finally, suppose, however, that we understand H to be an insufficient but nonredundant condition which is part of a set of conditions which is sufficient but not necessary for K. (Here H is an *INUS* cause of K. See J. L. Mackie, 1974.) Also, let J be an INUS cause. Then results similar to these for probabilistic causation can be obtained. For example, it can be shown that the correlation between H and K can be positive, negative, or zero, depending on the background features. To alter that case slightly, suppose again that H is not a *probabilistic* cause of K but it is a nonredundant condition which is part of a set of conditions which is a probabilistic cause of K but which is not necessary for K. (Here H is an *INUP* cause of K.) Then it can be shown, for example, that the correlation between H and K can be positive, negative, or zero, depending on the background features.[7]

We have noted that some writers hold the position that if A is a cause of B then A and B will be positively correlated, while other writers hold the position that if A is a cause of B then the correlation between A and B will determine the magnitude of A's causal influence on B. The results presented here show that each of these positions needs to be qualified, for the conception of causation being used and the background assumptions about the features of the other causes of B play important roles in determining the sign and the magnitude of the correlation between A and B. If A is a sufficient cause of B, or if A is a probabilistic cause of B (with probability 1), then in each situation considered above the correlation between A and B must be *non-negative*, but in general it will not equal 1. If A is a probabilistic cause of B (with probability $p < 1$), the sign and magnitude of the correlation will vary depending on the kind of situation. If A and the other causal factor are uncorrelated, the correlation between A and B will be *nonnegative*, but in general it will not equal p. If the expectation of A and other causal factor is zero, or if A and the other factor are maximally negatively correlated, the correlation between A and B can be *negative*. We also claimed that the INUS conception of causation and the INUP conception of causation produce results which parallel those obtained for the probabilistic conception.

4. PARTIAL CORRELATIONS, SCREENING OFF, AND PARTITIONING

Many scientists and philosophers maintain that there are general criteria for distinguishing between those situations in which A is a cause of B and B is a cause of C (where B is an intervening causal variable between A and C) and those situations where A is a common cause of B and C. These criteria supposedly would allow researchers to distinguish between basically different causal systems involving three variables. We refer to these general criteria as kinds of "partial correlation". Here we explicate the relations among the conception of partial correlation used by P. Kendall and P. F. Lazarsfeld, the conception of partial correlation used by H. B. Asher, H. M. Blalock, and H. A. Simon, the conception of screening off used by H. Reichenbach, the conception of partitioning used by P. Suppes, and finally the measures of causal influence which have been suggested by the advocates of path analysis and structural equation methods. In section 5 we offer some criticisms of these approaches to causal inference.

These conceptions of partial correlations are used as tests of certain causal hypotheses. For example, Kendall and Lazarsfeld (1950) argue that if A causes B and B causes C, or if B is a common cause of A and C, then certain partial correlations should be zero. In the latter case, A is said to be a *spurious* cause of C (p. 185). The partial correlations must vanish, or otherwise the hypothesis of an intervening cause (or a common cause) is disconfirmed (pp. 153–8). (Our intent here does not require that we discuss in detail the other conditions which the various authors require for causal hypothesis testing.)

It is helpful to introduce the following terminology and definitions. Let *Criterion I* (C_I) be $[P(A.C/B) - P(A/B)P(C/B)]$ and let *Criterion II* (C_{II}) be $[P(A.C/\bar{B}) - P(A/\bar{B})P(C/\bar{B})]$. (We assume $1 > P(B) > 0$.) As we will show, criteria C_I and/or C_{II} play central roles in each of the four conceptions explicated here.

Kendall and Lazarsfeld (1950) define the partial correlation between A and C with B held constant as:

$$(2) \qquad d_{AC,B} = P(A.B.C)P(B) - P(A.B)P(B.C).$$

Similarly, the partial correlation between A and C with $\bar{B}$ held constant is defined as:

$$(3) \qquad d_{AC,\bar{B}} = P(A.\bar{B}.C)P(\bar{B}) - P(A.\bar{B})P(\bar{B}.C).$$

Each of these expressions can be rewritten, given that $1 > P(B) > 0$, in the following form:

$$(4) \qquad d_{AC,B} = [P(A.C/B) - P(A/B)P(C/B)][P(B)]^2.$$

$$(5) \qquad d_{AC,\bar{B}} = [P(A.C/\bar{B}) - P(A/\bar{B})P(C/\bar{B})][P(\bar{B})]^2.$$

Thus, $d_{AC,B} = 0$ if and only if criterion C_I equals zero; $d_{AC,\bar{B}} = 0$ if and only if criterion C_{II} equals zero.

Kendall and Lazarsfeld use the concepts of *intervening causal vari-*

able and *common cause*. Reichenbach, on the other hand, uses the concepts of *causal betweeness* and *conjunctive fork*. According to Reichenbach (1956), an event B is causally between[8] A and C only if:

(6) $$P(C/(A.B)) = P(C/B).$$

In such a situation, B is said to *screen off* A from C. Given that $P(A.B) > 0$, as the definition of (6) requires, (6) can be rewritten as:

(7) $$P(A.C/B) = P(A/B)P(C/B).$$

Thus, (6) is satisfied if and only if C_I equals zero. Given that $P(A.B) > 0$, $d_{AC,B}$ equals zero if and only if B screens off A from C. It is important to note that Reichenbach does not require for B's being causally between A *and* C that $\bar{B}$ screen off A from C, while Kendall and Lazarsfeld do require that an intervening variable B must be such that B screens off A from C *and* that $\bar{B}$ screens off A from C.

Reichenbach also uses the concept of conjunctive fork to characterize the situation in which an otherwise improbable coincidence is explained by appeal to a common cause (Reichenbach, pp. 157–63.) In such a situation Reichenbach requires that the common cause B satisfy both of the following (and that all the correlations be positive):

(8) $$P(A.C/B) = P(A/B)P(C/B).$$

(9) $$P(A.C/\bar{B}) = P(A/\bar{B})P(C/\bar{B}).$$

Thus, (8) is satisfied if and only if C_I and $d_{AC,B}$ equal zero, while (9) is satisfied if and only if C_{II} and $d_{AC,\bar{B}}$ equal zero.

Suppes has developed the concepts of *prima facie cause*, *spurious cause*, *direct cause*, and *indirect cause* (1970, pp. 23–8). A is said to be a *prima facie cause* of C if A occurs before C and A is positively correlated to C. An event B is a *direct* cause of C if it is a prima facie cause of C and there is no *partition* D temporally between B and C which screens off B from C. A prima facie cause which is not direct is *indirect*. For example, suppose A prima facie causes B and B prima facie causes C. Then A is an indirect cause of C only if the *partition* (B, $\bar{B}$) screens off A from C: A is an indirect cause of C only if there exists (B, $\bar{B}$) such that (8) and (9) are satisfied. Suppes' nontemporal requirements for an indirect cause A of C by way of B are thus satisfied if and only if Kendall and Lazarsfeld's requirements for B's being an intervening variable between A and C are satisfied. On the other hand, Reichenbach's requirements (6) for causal betweeness is the same as (8), but he does not require (9).

Suppes also uses the concept of a *spurious* cause: an event A is a spurious cause of C if it is a prima facie cause of C and it is screened off from C by a *partition* of events $(B, \bar{B})$ which occur earlier than A. A *genuine* cause is a prima facie cause which is not spurious. Thus, A is a spurious cause of C only if there is a partition $(B, \bar{B})$ earlier than A such that (8) and (9) are satisfied. Suppes' nontemporal requirements for A's being a spurious cause of C with respect to the partition $(B, \bar{B})$ are satisfied if and only if Kendall and Lazarsfeld's requirements for B's being a common cause of A and C (and hence A's being a "spurious" cause of C) are satisfied, which in turn are satisfied if and only if Reichenbach's requirements for B's being a conjunctive fork (or common cause) of A and C are satisfied.

Let us now consider the conception of partial correlation that is used by Simon and Blalock. Simon and Blalock hold that if B is an intervening variable between A and C, or a common cause of A and C, then the partial correlation between A and C, with B "held constant" should be zero. The partial correlation is defined as:

(10) $$\rho_{AC,B} = [\Phi_{AC} - \Phi_{AB} \cdot \Phi_{AC}]/[(1 - (\Phi_{AB})^2)(1 - (\Phi_{BC})^2)]^{1/2}.$$

Given that Φ_{AB} and Φ_{BC} are less than one.

(11) $$\rho_{AC,B} = 0 \text{ if and only if } \Phi_{AC} = \Phi_{AB} \cdot \Phi_{BC}.$$

We have recently derived the following results (the details of the proof of the relations among C_{I}, C_{II}, and $\rho_{AC,B} = 0$ appear in Appendix C, which is available upon request):

If C_{I} and C_{II} both equal zero, then

(12) $$\rho_{AC,B} = 0.$$

If C_{I} and C_{II} have different signs, then there are cases where $\rho_{AC,B} \neq 0$ but also cases where

(13) $$\rho_{AC,B} = 0.$$

We can see from (12) that when Kendall and Lazarsfeld's requirements, Reichenbach's conjunctive fork requirements, and Suppes' requirements are satisfied, then the Simon-Blalock partial correlation is also zero. Equation (13), however, shows that $\rho_{AC,B} = 0$ does not entail that C_I and C_{II} both equal zero.

For variables which are interval-quantitative, defenders of path analysis and structural equation approaches argue that there are cases where B is a common cause (or B is an intervening variable), but where $\rho_{AC,B}$ does not go to zero. They suppose that B is a common cause of A and C, but that A is also a direct cause of C. Since B is an indirect cause of C by way of A, B is both a direct and an indirect cause of C. (See, for example, Asher, pp. 35–44; Bentler (1980); Duncan, 29–30; and our Appendix C.) Furthermore, they argue that the appropriate *quantitative* measure of A's causal influence on C is

$$(14) \qquad (\sigma_C/\sigma_A)\cdot(\rho_{AC} - \rho_{AB}\cdot\rho_{BC})/(1 - \rho_{AB}^2).$$

Here ρ_{AC} is the Pearson product-moment correlation, and σ_A^2 is the variance of A. In section 5 we consider the adequacy of this measure for dichotomous variables where phi correlations replace the Pearson product-correlations. In such a situation, the measure (14) would have the same sign as $\rho_{AC,B}$.

5. "PARTIAL CORRELATION" AS A MEASURE OF CAUSAL INFLUENCE

Some writers hold the position that if B is an intervening causal variable between A and C, or if B is a common cause of A and of C, then a certain "partial correlation" will go to zero; other writers maintain that a certain measure will give, say, the *magnitude* of the degree of B's causal influence on C. Our major conclusion here is that whether one of these "partial correlations" can even provide an adequate (qualitative) measure of one variable's causal influence primarily depends upon certain background assumptions concerning the other, outside causal factors and upon the relations among the causal variables in the system itself.

Suppose that A is a probabilistic cause of B (with probability $\frac{1}{2}$) and that B is a probabilistic cause of C (with probability $\frac{1}{2}$), where the outside cause V of B, outside cause W of C, and A are not causally related but where the pairwise mathematical expectations among A, V and W are zero. Then the partial correlation $\rho_{AC,B}$ can be negative, and hence the structural equation measure given in equation (14) can be negative, even though A has indirect but no direct causal influence on C.[9] With this background assumption if A is only a common probabilistic cause of B and C, the partial correlation $\rho_{BC,A}$ can be negative, and hence the structural equation measure can be negative, too. Since the partial correlation is not equal to zero, it follows by equation (12) that C_{I} and C_{II} are not both zero.

Let us change the background assumption so that A, V and W are causally unrelated and pairwise uncorrelated. It will be helpful to develop a characterization of the general causal system that will be considered in the remainder of this section. First, we use the non-negative numbers a, b, c, and d, whose sum equals one, to represent A's *direct* probabilistic causal influence on B and C in the situation S where the causes V and W are not present and where B has no *direct* causal influence on C. Using conditional probabilities, we let $P(B.C/A.S) = b$, $P(B.\bar{C}/A.S) = d$, $P(\bar{B}.C/A.S) = a$, and $P(\bar{B}.\bar{C}/A.S) = c$. If we now suppose that B *does* have a direct probabilistic causal influence on C, where f represents this magnitude and equals $P(C/B.\bar{A}.\bar{W})$, then it can be shown that $P(B.C/A.\bar{V}.\bar{W}) = b + df$, $P(B.\bar{C}/A.\bar{V}.\bar{W}) = d\cdot(1-f)$, $P(\bar{B}.C/A.\bar{V}.\bar{W}) = a$, and $P(\bar{B}.\bar{C}/A.\bar{V}.\bar{W}) = c$. (We also suppose that $P(B/\bar{A}.V) = 1$ and that $P(C/\bar{A}.\bar{B}.W) = 1$).

Given this causal system and this background assumption, the relevant equations for the various measures can be derived. (These equations appear in Appendix A, which is available upon request.) First, it can be shown that Blalock is correct to hold that if B is an intervening probabilistic causal variable between A and C, where A is not a direct cause of C, the partial correlation $\rho_{AC,B}$ equals zero, and hence the structural equation measure equals zero as well. On the other hand, Blalock is mistaken to claim that if A is a common probabilistic cause of B and C, where B is not a probabilistic cause of C, then $\rho_{BC,A} = 0$. For $\rho_{BC,A}$ can be positive, zero, or negative, and hence it cannot serve as a qualitative or a quantitative measure of B's causal influence on C. Using equation (12), it follows that C_{I} and C_{II} need not both equal zero.[10]

Additional difficulties arise when A is a probabilistic cause of B, B is a probabilistic cause of C, and A is a probabilistic cause of C – that is, when A is both a direct and an indirect probabilistic cause of C. Several writers hold the position that $\rho_{BC,A}$ can serve as an adequate measure, while others hold that the structural equation measure can serve as a measure of B's causal influence on C. But both positions are unjustified. Suppose that A is the sole cause of B and that the effect C has only two causes, A and B. Suppose, also, that B is either a sufficient cause of C or that B is a probabilistic cause of C with probability 1. First, where A is a probabilistic cause of B and C but never causes B and C at the same time, neither the partial correlation $\rho_{BC,A}$ nor the structural equation measure can give the answer that B's causal influence on C equals 1. Second, where A is a probabilistic cause of B and an (independent) probabilistic cause of C, neither of these measures can give the result that B's causal influence on C equals 1.[11] On the other hand, if we also suppose that A is a sufficient cause of B and C, and that there are other causes of B and C, then the partial correlation $\rho_{AC,B}$ will be zero even though A is sufficient for C, and the structural equation measure (partialling on A) will be positive but less than 1 even though B is sufficient for C.

Let us now suppose that B is a probabilistic cause of C with probability $\frac{1}{2}$ and that A is the only cause of B, while A and B are the only causes of C. Also, assume that A is a probabilistic cause of B and C with probability $\frac{1}{2}$ in both cases, but that A never causes B and C on the same occasion.[12] Then $\rho_{BC,A}$ and the structural equation measure will be negative. If the assumption about B's causal influence on C is held fixed and the other parameters of the system are varied, cases can be generated where the measure becomes positive, or negative, or zero. Similar findings are obtained when one considers situations where there are other outside causes of B and C. For example, if B is a probabilistic cause of C with probability $\frac{1}{2}$, then $\rho_{BC,A}$ and the path analysis or structural equation measure can be negative.[13,14] Given equation (12), it follows that C_{I} and C_{II} are not both equal to zero.

These results show that for a causal system which involves probabilistic causation it is an important assumption whether the outside causal factors are causally unrelated and pairwise uncorrelated. For where the pairwise mathematical expectations of the outside causal factors A, V, and W are zero, and where A is only an indirect cause of C by way of B or where A is only a common cause of B and of C, the relevant partial correlation and the relevant structural equation measure can be negative. Furthermore, even when it is assumed that the outside causal factors are causally unrelated and pairwise uncorrelated, difficulties arise. Where A is a probabilistic cause of B and C, but B is not a probabilistic cause of C, the relevant partial correlation and structural equation measure can be positive, zero, or negative. Where A is a probabilistic cause of B and C, and B is a probabilistic cause of C, the relevant partial correlation and the path analysis or structural equation measure can be negative. In such cases these expressions cannot serve as adequate measures of, say, B's causal influence on C. Thus, the background assumptions about the outside causal factors and about the relations among the variables in the causal system itself have important bearings on whether one of the general "partial correlation" measures can yield an adequate result. Although we have developed our arguments using probabilistic causation, similar results can be obtained for INUS causation.

6. SUMMARY

Philosophers and scientists maintain that causation, correlation, and partial correlation are essentially related. Many argue that the essential relation between causation and correlation enables one to use correlation as a measure of one variable's causal influence on another; many maintain that the essential relation between causation and partial correlation enables one to use partial correlation as a measure of one variable's causal influence on another. We have considered the claims of several philosophers and social scientists for causal systems with dichotomous variables.

In section 2 we explicated four conceptions of correlation, and in section 3 we concluded that none of them provides an adequate quantitative measure and that whether correlation can provide an adequate qualitative measure depends upon certain background assumptions about the outside causes. In section 4 we explicated five conceptions of "partial correlation", and in section 5 we concluded that none of them provides an adequate quantitative measure and that whether one of them can provide an adequate qualitative measure primarily depends upon certain background assumptions about the outside causal factors and the relations among the variables in the system itself.

The important conclusion is that the commonly used approaches (considered here) to causal hypothesis testing are in need of serious qualification and revision.

NOTES

* The authors contributed equally to this article. We also would like to acknowledge the support of the Spencer Foundation in the preparation of this manuscript.

1 Salmon claims that when A is a probabilistic cause (with probability p) of B, A need not be positively correlated with B. In section 3, after distinguishing between a sufficient and a probabilistic cause, we elaborate the claim and qualify it for cases where $p = 1$.

2 The measure of causal influence used by Asher and several others is a function of phi, the so-called beta coefficient (1976, pp. 36–7): Φ_{AB} times the standard deviation of B, σ_B, divided by the standard deviation of A, σ_A." See H. Blalock (1964, pp. 32, 71–7) and J. A. Davis (1975). (See also appendices A and B, equations A1 and B4. These appendices are available upon request.) For the dichotomous case the term is equivalent to the correlation of equation (c) above.

3 Due to the rather forbidding mathematical style and the complexity of the views of I. J. Good (1961–2), we consider them in a separate paper. (Authors, 'On Good's Causal Calculus' (in preparation.) There we argue, among other things, that Good's general view that A is a cause of B only if A is positively correlated with B (see his definition of a causal chain, II, p. 45, theorems 1, 2, and axiom 5, I, p. 310) is inadequate because there are situations where A is a cause of B even though A is not positively correlated with B.

4 This use of 'probabilistic causation' needs to be distinguished from 'INUP causation'. H is said to be an *INUP* cause of K when it is not a probabilistic cause of K but is a nonredundant condition which is part of a set of conditions which is a probabilistic cause of K but which is not necessary for K. Thus, the distinction between a probabilistic cause and an INUP cause parallels the distinction between a sufficient condition cause and an INUS cause, where H is said to be an *INUS* cause of K if H is an insufficient but nonredundant condition which is part of a set of conditions which is sufficient but not necessary for K. We shall discuss INUS causation later on in this section.

5 Of course, the statement "A causes B" may well mean other things as well, things associated with, say, causal priority and temporality. See, for example, Mackie (1974).

6 When H and J are uncorrelated, it can be shown that $P(H.K)P(\bar{H}.\bar{K}) - P(H.\bar{K})\cdot P(\bar{H}.K)$ equals $P(1-P)(1-rQ)p$, where $P(H) = P$, $P(J) = Q$ and $P(K) = pP + rQ - prPQ$. Given these results it is easy to show that none of the conceptions of correlation yield a plausible and an intuitive value when H is either a sufficient or a probabilistic cause of K. See also note 7.

7 For an INUS cause (or an INUP cause) it is relatively difficult to find an intuitive and plausible quantitative measure of H's causal influence on K. Suppose that $H.X$ is sufficient for K and that the only other (sufficient) cause is J. Then, if we use the ideas set out earlier in this section, $P(K/H.X.\bar{J}) = 1$ and $P(K/\overline{(H.X)}.\bar{J}) = 0$, but it also follows that $P(K/H.\bar{X}.\bar{J}) = 0$. It is rather easy to give intuitive meaning to the causal influence of $H.X$ on K, but it is difficult to do so for H by itself.

8 Reichenbach, p. 190. Reichenbach adds other conditions to his definition of *causal betweenness* that, for example, restrict the signs of the various correlations to be positive. These other conditions are independent of our main concern so we have ignored them.

9 When the pairwise expectations among A, V, and W are zero, let $P(A) = \frac{1}{2}$, $P(V) = \frac{5}{16}$, $P(W) = \frac{1}{6}$, and let V be a probabilistic cause ($\frac{4}{5}$) of B and W be a probabilistic cause ($\frac{1}{2}$) of C. Then $\Phi_{AC} - \Phi_{AB}\cdot\Phi_{BC}$ is negative. See Ellett and Ericson (1983).

10 Using the ideas presented above, the first example takes $a + b = 0$, from which it follows $\Phi_{AC} = \Phi_{AB}\cdot\Phi_{BC}$. The second example takes $f = 0$, from which follows that $\Phi_{BC} - \Phi_{AB}\cdot\Phi_{AC}$ equals the sign of $b - (a+b)(b+d)$. When $b = (a+b)(b+d)$, then C_{I} and C_{II} are both zero.

[11] The first example takes $P(A) = \frac{2}{3}$, $P(V) = P(W) = 0$, $f = 1$, and $b = c = 0$. The second example takes $P(A) = \frac{2}{3}$, $P(V) = P(W) = 0$, $f = 1$, and $b = (a + b)(b + d) > 0$.

[12] The example takes $P(A) = \frac{2}{3}$, $P(V) = P(W) = 0$, $f = \frac{1}{2}$, $a = d = \frac{1}{2}$, and $b = c = 0$.

[13] The example takes $P(A) = \frac{2}{3}$, $P(V) = P(W) = \frac{1}{10}$, $f = \frac{1}{2}$, $a = d = \frac{8}{20}$, and $b = c = \frac{1}{10}$.

[14] In appendix D, available upon request, a brief critical discussion of the relevant measure used by log-linear models of analysis is given.

REFERENCES

Asher, H. B.: 1976, *Causal Modeling*. Sage Publications, Beverly Hills.

Bentler, P. M.: 1980, 'Multivariate Analysis with Latent Variables; Causal Modeling', *Annual Review of Psychology* **31**, 419–56.

Blalock, H. M., Jr.: 1964, *Causal Inference in Nonexperimental Research*, University of North Carolina Press, Chapel Hill.

Campbell, D. T. and J. C. Stanley: 1963, *Experimental and Quasi Experimental Designs for Research*, Rand McNally, Chicago.

Cartwright, N.: 1979, 'Causal Laws and Effective Strategies', *Nous* **13**, 419–37.

Cartwright, N.: 1983, *How the Laws of Physics Lie*, Oxford University Press, Oxford.

Davis, J. A.: 1975, 'Analyzing Contingency Tables with Linear Flow Graphs: D-Systems', in D. R. Heise, (ed.), *Sociological Methodology*, Jossey-Bass Publications, San Francisco, 1976, pp. 111–45.

Duncan, O. D.: 1975, *Introduction to Structural Equation Models*, Academic Press, New York.

Ellett, F. S., Jr. and D. P. Ericson: 1983, 'On the Logic of Causal Methods in Social Science', *Synthese* **57**, 67–82.

Ellett, F. S., Jr. and D. P. Ericson: 1985, 'Causal Laws and Laws of Association', *Noûs* **19**, no. 4.

Good, I. J.: 1961–1962, 'A Causal Calculus I–II', *British Journal for the Philosophy of Science*, Vol. XI, No. 44, pp. 305–18; Vol. XII, No. 45, pp. 43–51.

Kendall, P. L. and P. F. Lazarsfeld: 1950, 'Problems of Survey Analysis', in R. K. Merton and P. F. Lazarsfeld (eds.), *Continuities in Social Research: Studies in the Scope and Methods of "The American Soldier"*, Free Press, Glencoe, Ill, pp. 133–96.

Mackie, J. L.: 1974, *The Cement of the Universe*, Oxford University Press, Oxford.

Nagel, E.: 1961, 'Explanation and Understanding in the Social Sciences', in *The Structure of Science*, Harcourt, Brace, and World, New York, Chapter 14.

Reichenbach, H.: 1956, *The Direction of Time*, University of California Press, Berkeley.

Salmon, W. C.: 1980, 'Probabilistic Causality', *Pacific Philosophical Quarterly*, Vol. 61, Nos. 1 and 2, Jan–April, pp. 50–74.

Simon, H. A.: 1957, 'Spurious Correlation: A Causal Interpretation', in *Models of Man*, Wiley, New York, Chapter 3.

Suppes, P.: 1970, *A Probabilistic Theory of Causality*, North-Holland, Amsterdam.

Graduate School of Education
University of California
Los Angeles, CA 90024
U.S.A.